Code: 19BS1402

II B.Tech - II Semester – Regular Examinations – AUGUST 2021

ENGINEERING MATHEMATICS – IV
(Probability Theory and Random Processes)
(ELECTRONICS AND COMMUNICATION ENGINEERING)
(Gaussian distribution table-Appendix B will be provided)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 5 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each question carries 12 marks.
4. All parts of Question paper must be answered in one place

PART – A

1. a) Mention the axioms of probability.
b) Let X is a continuous random variable with PDF $f_X(x) = \frac{8}{x^3}$, $x > 2$. Find E[w] where w=X.
c) State central limit theorem.
d) What are all the conditions for Random processes to be a jointly wide sense stationary?
e) State the properties of Power Spectral Density.

PART – B

UNIT – I

2. a) A lot of 100 semiconductor chips contain 20 that are defective. Two chips are selected at random, without replacement, from the lot. i) What is the probability that the first one selected is defective? ii) What is the probability that both are defective? 6 M
b) In a game of dice, a "shooter" can win outright if the sum of the two numbers showing up is either 7 or 11 when two dice are thrown. What is his probability of winning outright? 6 M

OR

3. a) A student is known to arrive late for a class 40% of the time. If the class meets five times each week find (i) the probability the student is late for at least three classes in a given week, and (ii) the probability the student will not be late at all during a given week. 6 M

b) In a box there are 80 resistors, each having the same size and shape. Of the 80 resistors, 18 are 10Ω, 12 are 22Ω, 33 are 27Ω and 17 are 47Ω. If the experiment is to randomly draw out one resistor from the box with each one being "equally likely" to be drawn, find the following conditional probabilities: P(draw 10Ω/22Ω), P(draw 22Ω/22Ω), P(draw 27Ω/22Ω) and P(draw 47Ω/22Ω). 6 M

UNIT – II

4. a) A continuous random variable X has a probability density function: $f(x) = 3x^2$ for $0 \leq x \leq 1$.
Find 'a' and 'b' such that $P\{x \leq a\} = P\{x > a\}$ and $P\{x > b\} = 0.05$ 6 M

b) A random variable X is known to be Gaussian with a_X=1.6 and σ_X=0.4. Find a) P(1.4<X≤2) b) P(-0.6<(X-1.6) ≤0.6). (Note Use Gaussian distribution table) 6 M

OR

5. a) (I) Verify that the function p(x) defined by

$$p(x) = \begin{cases} \frac{3}{4}\left(\frac{1}{4}\right)^x & x = 0,1,2,3.. \\ 0 & otherwise \end{cases}$$

is a probability mass function of a discrete random variable X.
(II) Find i) P(X = 2) ii) P(X ≤2) iii) P(X ≥1). 6 M

b) The number of cars arriving at a certain bank drive-in window during any 10 min period is a Poisson random variable X with b=2. Find i) The probability that more than 3 cars will arrive during any 10 min period.
ii) The probability that no cars will arrive. 6 M

UNIT-III

6. a) Develop the relation ${\sigma_X}^2 = m_2 - {m_1}^2$. 6 M

b) The joint density function of two random variables X and Y is $f_{XY}(x,y) = u(x)u(y)\frac{1}{12}e^{-(\frac{x+y}{4})}$.
Check the independence between X and Y. 6 M

OR

7. a) A random variable has a probability density

$$f_X(x) = \begin{cases} \left(\frac{5}{4}\right)(1 - x^4) & 0 < x \leq 1 \\ 0 & elsewhere \end{cases}$$ 6 M

Find: i) $E[X]$ ii) $E[4X + 2]$ and iii) $E[X^2]$.

b) Given the function

$$f_{x,y}(x,y) = \begin{cases} b(x+y)^2, -2 < x < 2 \text{ and } -3 < y < 3 \\ 0, \quad elsewhere \end{cases}$$

(i) Find the constant b such that this is a valid joint density function.
(ii) Determine the marginal density functions $f_X(x)$ and $f_Y(y)$. 6 M

UNIT – IV

8. a) Show that the random process $X(t) = Acos(\omega_0 t + \theta)$is a wide sense stationary. If it is assumed that A, $\omega_0$are constants and θ is a variable uniformly distributed over the interval $(0,2\pi)$ 6 M

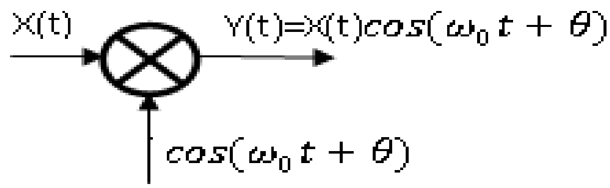
b) Assume that an ergodic random process X(t) has an autocorrelation function

$$R_{XX}(\tau) = 18 + \frac{2}{6+\tau^2}[1 + 4\cos(12\tau)]$$

i) Find$|\bar{X}|$ ii) Does this process have a periodic component iii) What is the average power in X(t)? 6 M

OR

9. a) Let X(t) be a wide sense stationary process with the auto correlation function $R_{XX}(\tau) = e^{-a|\tau|}$, a > 0. Assume $X(t)$ amplitude modulates a carrier $cos(\omega_0 t + \theta)$ as shown in the figure below. Here θ is a variable uniformly distributed on $(-\pi, \pi)$. Show that $Y(t)$ is a wide sense stationary process 6 M



b) Differentiate between the following:
i) Time averages and statistical averages.
ii) Mean ergodicity and auto correlation ergodicity. 6 M

UNIT – V

10. a) A random process $X(t) = Acos(\omega_0 t + \theta)$, where A, ω_0 are constants and θ is a variable uniformly distributed on the interval$(0,2\pi)$. Find the average power? 6 M

b) Assume a random process has a power spectrum

$$S_{XX}(\omega) = \begin{cases} 4 - (\omega^2/9), |\omega| \le 6 \\ 0, elsewhere \end{cases}$$

Find (i) the average power, (ii) the RMS bandwidth, and (iii) the autocorrelation function of the process 6 M

OR

11. a) Develop the relation between cross-correlation function and cross power spectral density and comment on it. 6 M

b) Find the PSD of a random process $Z(t) = X(t) + Y(t)$ where $X(t)$and $Y(t)$ has zero means, individual random process and $X(t)$and $Y(t)$ are also jointly wide sense stationary processes. 6 M